$w = v + \alpha u$

Q. Are the function $u = \frac{x-y}{x+z}$, $v = \frac{x+z}{y+z}$ functionally dependent? If so find the relation between them?

Sol^n.
$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{y+z}{(x+z)^2} & \frac{-1}{(x+z)} \\ \frac{1}{(y+z)} & -\frac{(x+z)}{(y+z)^2} \end{vmatrix}$$

$$= \frac{1}{(y+z)^2(x+z)^2} \begin{vmatrix} y+z & -(x+z) \\ y+z & -(x+z) \end{vmatrix}$$

$$= \frac{-1}{(y+z)^2(x+z)^2} (y+z)(x+z) \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$\dots + my + nz = 0$ and $\dots + \dots + \dots = 1$ are the roots $\dots$

$$\frac{\partial(u,v)}{\partial(y,z)} = \begin{vmatrix} -\frac{1}{(x+z)} & -\frac{x-y}{(x+z)^2} \\ -\frac{x+z}{(y+z)^2} & -\frac{(x-y)}{(y+z)^2} \end{vmatrix}$$

$$= \frac{1}{(x+z)^2(y+z)^2} \begin{vmatrix} -(x+z) & -(x-y) \\ -(x+z) & -(x-y) \end{vmatrix}$$

$$= \frac{(x+z)(x-y)}{(x+z)^2(y+z)^2} \begin{vmatrix} -1 & -1 \\ -1 & -1 \end{vmatrix} = 0$$

Similarly $\frac{\partial(u,v)}{\partial(x,z)} = 0$

So they are not independent.

$$1-u = 1-\frac{x-y}{x+z} = \frac{x+z-x+y}{x+z} = \frac{y+z}{x+z} = \frac{1}{v}$$

or $v = \frac{1}{1-u}$

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Q. If $u = ax^2 + by^2 + cz^2$ where $x^2+y^2+z^2=1$ and $lx+my+nz=0$ prove that the stationary ~~point~~ values of u satisfy the eqⁿ $\frac{l^2}{a-u} + \frac{m^2}{b-u} + \frac{n^2}{c-u} = 0$

Solⁿ $F(x,y,z) = (ax^2+by^2+cz^2) + \lambda(x^2+y^2+z^2-1) + \mu(lx+my+nz)$.

diff Partially wrt x, y & z

$2ax + 2\lambda x + l\mu = 0$ —(1)

$2by + 2\lambda y + m\mu = 0$ —(2)

$2cz + 2\lambda z + n\mu = 0$ —(3)

eq (1) $\times x$ + eq (2) $\times y$ + eq (3) $\times z$

$2(ax^2+by^2+cz^2) + 2\lambda(x^2+y^2+z^2) + \mu(lx+my+nz) = 0$

$2u + 2\lambda + 0 = 0$

$\lambda = -u$

from (1) $2ax - 2ux + \mu l = 0$

$\mu l = 2x(u-a)$ $\quad x = \frac{\mu l}{2(u-a)}$

similarly $y = \frac{\mu m}{2(u-b)}$, $z = \frac{\mu n}{2(u-c)}$

Now

$lx+my+nz = \mu\left(\frac{l^2}{2(u-a)} + \frac{m^2}{2(u-b)} + \frac{n^2}{2(u-c)}\right)$

$lx+my+nz = 0$

$\therefore \frac{l^2}{u-a} + \frac{m^2}{u-b} + \frac{n^2}{u-c} = 0$ Ans

Q. The temp. T at any point (x, y, z) in space is $T = 400xyz^2$. find the highest temp. at the surface of a unit sphere $x^2+y^2+z^2=1$

Sol^n $F(x,y,z) = 400xyz^2 + \lambda(x^2+y^2+z^2-1)$ —— (1)

$dF = 0$

$400yz^2 + 2\lambda x = 0$ —— (2)

$400xz^2 + 2\lambda y = 0$ —— (3)

$\cancel{400}\ 800xyz + 2\lambda z = 0$ —— (4)

(2) $\times \cancel{x}$ + eq (3) $\times y$ + eq (4) $\times z$

$1600xyz^2 + 2\lambda(x^2+y^2+z^2) = 0$

$\{x^2+y^2+z^2=1\}$

$\lambda = -800xyz^2$

from (2)

$400yz^2 - 1600x^2yz^2 = 0 \Rightarrow x = \pm\frac{1}{2}$

Similarly $y = \pm\frac{1}{2}$, $z = \pm\frac{1}{\sqrt{2}}$

max.

$T = 400xyz^2$

$= 400 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 50$ Ans

Q. Find the volume of the largest parallelopiped with edges parallel to the axes that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Soln $F(x,y,z) = 8xyz + d\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1\right)$ — (1)

where $2x, 2y, 2z$ be the length, breadth & height of ellipsoid $V = 8xyz$

$$dF = 0$$

$8yz + d\left(\frac{2x}{a^2}\right) = 0 \Rightarrow d = -\frac{4yz \cdot a^2}{x}$

$8zx + d\left(\frac{2y}{b^2}\right) = 0 \Rightarrow d = -\frac{4zx \cdot b^2}{y}$

$8xy + d\left(\frac{2z}{c^2}\right) = 0 \Rightarrow d = -4xy\,\frac{c^2}{z}$

on equating I & II $y^2 = \frac{b^2}{a^2} \cdot x^2$

I & III $z^2 = \frac{c^2}{a^2} x^2$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$\frac{x^2}{a^2}+\frac{x^2}{a^2}+\frac{x^2}{a^2}=1 \Rightarrow x = \frac{a}{\sqrt{3}}$

$y = \frac{b}{\sqrt{3}}$

$z = \frac{c}{\sqrt{3}}$

$$V = \frac{8abc}{3\sqrt{3}}$$